The "Risk" of Ignoring Risks in Retirement Financial Planning

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1. Summary

This paper explores the consequences of ignoring the risks associated with retirement financial planning. Using an affluent Canadian case example, we first present a conventional approach to drawing down retirement income. We then introduce mortality, financial market and inflation rate risk, and explore the implications. Finally, we investigate alternative drawdown strategies where the individual targets the same initial financial income flows but reduces overall risk by adjusting, throughout his or her lifetime, those factors that he/she can control.

We carry out our analysis using a longitudinal, comprehensive, dynamic and integrated micro-simulation model for Canada, incorporating stochastic inflation, financial markets, mortality and interactions among sources of retirement income within the complex Canadian tax and social benefit systems. This analysis uses the 2017 Projection Assumption Guidelines compiled by the Institut québécois de planification financière (IQPF) and the Financial Planning Standards Council (FPSC) (Bachand et al., 2017) as a key reference for mortality, expected returns, inflation and standard deviation.

Our findings show the dangers of relying on simplistic, deterministic and "stylized" examples when evaluating alternative financial strategies. Under a conventional retirement financial strategy (which aims to exhaust savings by age 90 by basing calculations on a single possible deterministic "future") a high net worth Canadian male faces a 17% probability of running out of money. For a high net worth Canadian female, the probability is 31%. Furthermore, at age 91, there is a 90% probability that his/her net income would fall by nearly 50%. Not only is this a negative outcome for the individual, but it also affects the broader Canadian economy since, after such a drop, the individual would become eligible to collect GIS benefits.

However, by choosing a risk-reducing strategy of adjusting withdrawals based on market performance – as well as delaying collecting CPP until age 70 and using RRSP savings to bridge the gap – the same person could achieve the same initial income and maintain control over accessible funds, while eliminating the risk of running out of savings. At the extreme, the individual could experience a decline in net income of 30% (22% at age 80, and an additional drop of 8% at age 89). Although drops in income are never desirable, research finds that seniors are much more adaptable to moderate drops than to severe drops, which can severely derail their retirement budget.

This risk-reducing strategy also requires no dependence on GIS benefits, ensures a continuing pool of accessible funds to protect against unanticipated financial shocks and other needs, and guarantees an

additional $5,100 in CPP income until death, transferring financial market risk, inflation risk or longevity risk.

Not only does ignoring risk lead to misleading – and possibly harmful – conclusions, the deterministic methodology cannot appropriately measure or assess the value of risk-reducing financial strategies. As a result, workers and retirees who base their retirement decisions on deterministic projections may miss out on strategies that provide the desired level of annual income while also offering protection from risks that can severely impact an individual's financial well-being at advanced, vulnerable ages.

If affluent Canadians can manage their savings to sustain their current consumption in retirement, this will protect the rest of Canadians against the cost and risk burdens of funding additional senior social programs (OAS and GIS) and the various provincial income-tested benefits. Today's 65-year-old Canadian has a 50% probability of living beyond age 90 (Bachand et al., 2017), and affluent individuals have even better longevity expectations. The percentage of elderly seniors as a proportion of the working Canadian population is expected to double from 2010 to 2030 (Statistics Canada, 2010). Helping retiring Canadians to get more clarity on their financial decisions and the underlying risks will benefit not just Canada's elderly population, but also the overall Canadian economy.

In broader terms, this research supports the practice of periodically adjusting financial strategies based on emerging information to help manage risk. For example, financial planners who rely on deterministic projections can still help mitigate some risk and help clients achieve their goals (such as not outliving their savings) by continually reviewing retirement financial strategies based on market performance, inflation, life expectancy and any other material changes in the client's circumstances.

The research also reinforces the importance of considering risk implications when reviewing key decision opportunities – for example, at what age to start collecting CPP benefit payments, whether to take the commuted value of a pension plan versus the actual pension income payments, or when and how much to tap into home equity. Appreciating that deterministic projections cannot appropriately measure, and therefore not assess, the value of such risk-reducing financial choices, an individual or advisor who relies on a deterministic approach will need to adopt additional analysis methods to more fully explore the risk implications of alternative futures.

Our findings support the importance of treating financial planning as an ongoing, comprehensive and integrated problem that includes an individual's full financial circumstances and preferences, the complex Canadian system of taxes and social benefits, and the current financial environment and spectrum of potential risks.

2. Introduction and Background

Financial planners and actuaries regularly identify sources of risk relevant for retirement financial planning. Such sources include, for example, labour market risk, investment returns risk, inflation risk, living 'too long' in the sense of outliving one's financial resources (longevity risk), and the unexpected costs associated with health shocks, as well as other financial shocks (such as divorce, widowhood, and unexpected home renovations/repairs). The relatively long time spans inherent in individual financial strategies offer major opportunity for these risks to generate significant, interacting, cumulative impacts on the financial outcomes that people will experience.

Despite the widespread appreciation for the significant impacts of risk on financial outcomes, deterministic "one future" projections that are void of risk implications continue to dominate the financial planning landscape. This includes online financial planning software[1], books and magazine articles providing financial planning advice, and face-to-face financial planning counselling[2],[3].

The impact of mismanaging savings and not accounting for risk is a major concern for seniors who do not have adequate secure retirement income from workplace pensions, C/QPP, and OAS benefits, and rely on personal savings to sustain their consumption in retirement. While younger seniors generally prefer financial flexibility, advanced age seniors often require financial security – and unfortunately, "risk" operates in precisely opposite terms, as the elapse of time enables risk to have greater opportunity to generate significant, interacting, cumulative impacts. All else equal, financial strategies that ignore risk are progressively more likely to fail with time. As a result, an elderly senior is increasingly more likely to deplete financial savings as he/she advances in years[4]. While younger, healthier seniors are generally better able to absorb financial risks by adjusting spending, or even returning to the workforce to supplement any income shortfalls, seniors at later ages are more likely to be more financially, mentally, and physically vulnerable to income shortfalls. For example, it is at advanced ages (e.g. 85 and beyond) that inflation has eroded any nominally fixed pension income by nearly a third (2 percent compounded over 20 years), increased frailty and skills attrition has reduced the possibility of returning to the workforce, and cognitive declines expose vulnerable seniors to financial decision-making mistakes (Finke et al. 2017). In addition, the advanced-aged elderly are much more likely to have experienced the impacts of financial shocks associated with the onset of chronic health conditions, which creates fixed and ongoing health care costs that cannot be postponed without impact. With the population aging and a decline in secure employer pension plan income, more and more Canadians will face this reality.

Seniors with savings manage financial and longevity risk in two general ways: (1) by transferring the risk, which can be done formally through the purchase of insurance products (such as annuities), or informally through transfers of wealth and services traditionally done within the family; and/or (2) by self-managing the risk. Although data is limited, the literature is increasingly shedding insights on the

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[1] Including the Canadian government retirement income calculator: https://www.canada.ca/en/services/benefits/publicpensions/cpp/retirement-income-calculator.html

[2] In a study for the Society of Actuaries' second-round evaluation of financial planning software used by financial advisers and investors, Turner and Witte (2009, p. 10)2 noted "… the major conclusions of the first study still hold. Notably, most financial modeling programs do not do a good job of evaluating the risks that users face." They identified some of the critical issues that make financial planning challenging, but that are often missing from actual financial plans. Among others, they specifically highlighted the omission of taxes, inadequate consideration of government benefits, and simplistic treatments of labour force attachment, especially involuntary retirement.

[3] Moreover, simplistic deterministic financial analysis are often employed to demonstrate financial planning "rules of thumb", which include "save 10% of earnings to prepare for retirement," "withdraw 4% of savings in retirement," "replace 70% of pre-retirement earnings in retirement to preserve living standards." Not only lacking in empirical evidence and widely questioned, these rules of thumb overlook a person's financial circumstances and preferences, and ignore the risks that he/she faces. Rules of thumb provide no indication of the degree of variability that a person is likely to experience or how alternative financial strategies might affect the level of that expected variability. They do not indicate how to adjust savings during the accumulation period, or consumption during the drawdown period, to reflect the experience to date and expectations and preferences about the future.

[4] Otherwise known as "set and forget", fixed withdrawal drawdown strategies that do not respond to risk have been repeatedly found to have an especially high risk of depleting financial savings (see, for example, Blake et al. (2003)). For an extensive summary of the literature on drawing down financial savings, including fixed withdrawal strategies, see MacDonald et al. (2013).

actual strategy choices that seniors make. For example, voluntary annuitization is extremely rare. This has been the focus of a great deal of study (see MacDonald et al. (2013) for summary). Correspondingly, other than medical insurance, financial insurance products are not popular among seniors (SOA, 2016). In addition, the potential for risk sharing within the family is expected to decline considerably, particularly informal care services that families have traditionally provided to elderly members with chronic health care conditions[5].

On the other hand, a variety of reports are increasingly suggesting that seniors who self-manage assets in retirement attempt to control their risk exposures by choosing not to spend financial savings. For example, a comprehensive recent study found that, on average, American retirees drew 1% of assets per year and had more than 80% of assets even after two decades of retirement (Wolfe and Brazier, 2017). In fact, account balances grew for over a third of retirees (ibid)[6]. Society of Actuaries (SOA) provided insight into the motivation underlying these empirical findings. Building on a series of surveys and focus group studies by the Society of Actuaries (SOA) over the past 15 years that explore how workers and retirees perceive and respond to retirement risks, SOA (2016) found that seniors choose not to use their savings to protect against future risks. This precautionary behaviour, however, leads to significant unspent savings at the time of death and an unnecessarily reduced lifestyle.

Rather than not spend and benefit from their savings in retirement, seniors could alternatively self-manage risk by choosing withdrawal strategies that respond to risk. Known as "variable" drawdown strategies, seniors effectively manage investment risks by adjusting their annual payments to reflect experienced investment returns and portfolio composition. In exchange for fluctuating payments, dynamic self-managed strategies protect against running out of savings (see MacDonald et al. (2013) for an extensive review of supporting literature). This exchange appears to be consistent with the preferences and abilities of seniors, who have been found to have significant flexibility to make spending adjustments, but would prefer to hold onto assets and reduce overall spending rather than face the threat of derailing retirement plans when a risk-related event severely reduces income (SOA, 2016).

A second approach to self-managing risk is by taking advantage of key decision opportunities, such as the following choices:

- the age to begin collecting C/QPP benefit payments;
- directing funds to reducing debt or savings;
- renting versus owning primary residence; and
- the homeowner's option to draw on housing equity (either through the sale of the home or a reverse mortgage).

The purpose of this paper is to present the implications of ignoring risk, and test the effect of two risk-reducing strategies – namely (1) varying savings withdrawals in response to financial market returns and

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[5] For example, currently nearly three-quarters of all senior home care is done by family members (Health Council of Canada 2012), but seniors with chronic health conditions will increasingly need to pay for this costly expense themselves, because of smaller family sizes, greater mobility of family members, greater participation of women in the workforce, and changing expectations of care within families (Gibson and Houser 2007; Pickhard, 2008, 2011; Keefe et al. 2012).

[6] Other studies that came to similar conclusions include De Nardi et al. (2006), Love et al. (2008), Smith et al. (2009), and Poterba et al. (2011).

(2) delaying CPP take-up. This study focuses on three major sources of negative risk in retirement that can lead to exhausting savings prematurely:

1. Poor investment returns;
2. High inflation; and
3. Living longer than anticipated.

We carry out the analysis using as example an affluent Canadian who depends on personal savings to sustain retirement. The analysis begins by presenting what might be conventional financial planning advice. Then, to explore the implications of risk for retirement financial planning, we next integrate into the projection a stochastic modeling of inflation, financial market returns, and mortality. We finally test several alternative financial strategies intended to reduce risk while providing the targeted return.

The rest of this paper flows as follows: Section 3 outlines our methodology; Section 4 presents the results; and Section 5 contains our conclusions.

3. Methodology

3.1 Example Case Affluent Canadian

MacDonald et al. (2017) found that financial planning decisions are especially important for affluent retiring Canadians with a high level of personal savings and a low level of pension income. Such affluent Canadians could include professionals or small business owners who depend on defined contribution plans and their private financial wealth in retirement. We carry out this study, therefore, using the example of a single Canadian with significant financial savings and no employer pension plan. We present calculations for a 65-year old single Canadian male (we will also show gender impacts in the analysis) in 2014 living in Ontario, with:

- $500,000 in RRSP assets,
- $10,000 in net non-registered financial assets,
- $52,000 in TFSA assets,
- Maximum Canada Plan (CPP) benefits,
- No employer pension plan benefits

Income tax and OAS/GIS benefit are computed directly in the simulation (see Section 3.3). Holdings in excess of $500,000 in financial savings represents only affluent Canadian seniors and does not reflect Canadian seniors in general. Among Canadians between 60 and 70 years old in 2005, for example, 25% held over $100,000 in net financial savings, 16% held over $200,000, 11% held over $300,000 6.5% held over $400,000, and 5.9% over $500,000[7].

[7] Based on the 2005 SFS weighted data. We divide the household values reported in the SFS by the square root of the number of household members to arrive at single-adult-equivalent values. We calculate financial assets as the sum of registered financial assets (RRSP and RRIF) and non-registered financial assets (chequing accounts, GICs, trusts, etc), less non-mortgage debt (credit card, lines of credit, car loans, etc).

3.2 Drawdown Strategies

Our analysis focuses on the outcomes associated with alternative financial strategies that the subject might employ. This section describes the drawdown strategies and projection assumptions employed in this study.

3.2.1 Fixed drawdown strategy

Commonly known as the "4% Rule" (Bengen, 1994)[8], the fixed "Set and Forget" drawdown strategy sets a withdrawal amount at age 65, which remains constant each year (inflation adjusted). The conventional time horizon of financial planning is retirement to age 90, and therefore this study initially sets the withdrawal rate at 5%, which exhausts assets by age 90 based on a deterministic projection using our simulation assumptions (see Section 3.2.4)[9]. For comparison purposes, we also use a withdrawal rate of 3.3% - intended to represent a "lifetime" drawdown strategy, in that 3.3% is the calculated amount necessary to exhaust funds at the highest age on the mortality table (110).

Fixed Drawdown "5% Rule" (inflation-indexed)

- The individual withdraws 5% of the portfolio in the first year, and the same amount indexed by inflation in each subsequent year until death or portfolio exhaustion, represented by the following formula:

$$Drawdown_{65} = 0.05 Wealth_{6}$$
$$Drawdown_{x} = Drawdown_{x-1}(1 + Inflation_{x-1}) \text{ for } x > 65$$

where

- $Inflation_{x-1}$ is the rate of inflation between ages *x-1* and *x (see below for inflation modeling assumptions)*

3.2.2 Variable drawdown strategy

In a variable drawdown strategy, the drawdown amounts are dynamic in that they vary each year according to investment performance. In this analysis, the variable drawdown strategy targets to exhaust the funds at the highest age to which the individual may live (age 110 as noted above).

Variable "Dynamic" Lifetime Drawdown (the drawdown amount is re-calculated each year)

- The individual aims to withdraw, in an expected value sense, equal real amounts each year that exhaust the portfolio by age 110.

The variable strategy guarantees that funds will not be exhausted before death at the cost of fluctuating withdrawals. This is accomplished by recalculating the payments each year. The calculation is more easily understood by comparing it to the more common "1/T Rule", which essentially divides the total wealth

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[8] The 4% Rule is a popular self-managed drawdown strategy intended to protect against running out of funds before death. The origins of the 4% Rule can be traced to Bengen (1994), which argued that funds are likely not to run out under a 4% inflation-indexed fixed strategy (see also Pye (2000) and Ameriks et al. (2001)).

[9] The precise drawdown amount is 5.02%.

remaining by the number of years until desired portfolio exhaustion. For example, in the case of a 65 year-old who aimed to exhaust assets by age 110, the "1/T Rule" would set T= 45 (110 - 65 = 45). The drawdown amounts at ages 65, 80, and 109 (for example) would be:

- $Drawdown_{65} = Wealth_{65} / 45$ at age 65 (the denominator is 110 less 65);
- $Drawdown_{80} = Wealth_{80} / 25$ at age 80;
- $Drawdown_{109} = Wealth_{109} / 1$ at age 109; and
- $Drawdown_{110} = 0$ at ages 110 and over;

Because the "1/T Rule" does not incorporate anticipated future real portfolio returns, however, payments generally increase over time (Dus et al., 2004). To account for anticipated future asset returns so as to target level, inflation-indexed payments (that exhaust the portfolio at the end of the chosen horizon), the payments under the dynamic variable drawdown are calculated, using basic standard "International Actuarial Notation", as[10]:

$$Drawdown_x = Wealth_x \Big/ \ddot{a}_{\overline{110-x}|_r} \quad \text{for } x \in [65,109]$$

where $\ddot{a}_{\overline{y}|_r}$ is the actuarial present value of $1 (inflation-indexed) at the beginning of each future year for y years (e.g. inflation-indexed y-year annuity-certain)

- $\ddot{a}_{\overline{y}|_r} = \sum_{t=0}^{y-1} \frac{(1+m)^{-t}}{(1+n)^{-t}} = \sum_{t=0}^{y-1} (1+r)^{-t}$, where *m* is the assumed long-term mean inflation, *n* is the assumed long-term mean total nominal return on assets, and *r* is the assumed long-term mean total *real* rate of return on assets assumption

The actuarial present value factor is re-calculated each year using the anticipated long-term mean real rate of return on self-managed assets. The drawdown amount is only constant in real terms if the realized asset returns matched their projected average in each simulation year. Given the inevitable year-to-year fluctuations in actual returns, however, the payments are variable.

3.2.3 Delay CPP take-up

A risk-mitigating financial strategy is to delay CPP take-up from age 65 to age 70, and use RRSP savings to bridge the gap. Starting CPP benefits at age 70 instead of 65 will increase CPP pension by 42%. For example, if a 65 year-old is entitled to a CPP pension of $10,000 per year, he/she would get a pension equivalent to $14,200 a year if he/she delays taking the benefit until age 70. Despite the fact that it's broadly agreed that most seniors would benefit from more guaranteed fixed income[11], and delaying CPP is considered the cheapest and safest approach to securing annuitized income in retirement, very few

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[10] See Brown et al. (2015) and Dickson et al. (2013).

[11] A close reading of over 150 academic research papers on how best to draw down retirement savings confirms broad agreement among researchers and other experts that workers benefit greatly by annuitizing a significant portion of their savings at retirement (MacDonald et al., 2013). A wide spectrum of analysts with diverse backgrounds from different countries, using numerous approaches to measure success, have reached this conclusion. But voluntary annuitization is rare, which has been the focus of a great deal of study (ibid).

Canadians choose to delay their CPP take-up – in fact, the majority of Canadians take CPP before normal retirement age (65). There have been many articles from industry promoting the value of waiting to collect CPP benefits from age 65 to 70 – and a lot of mistrust and resistance from the public[12].

3.2.4 Simulation assumptions

When simulating the drawdown of wealth over the individual's retirement we use:

- 2007 gender-specific Canadian population mortality rates (Canada Life Tables)
- 2014 Canadian Government tax/benefit values assuming that 2011 rules, including those regarding indexation, extend into the future[13];
- Self-managed asset portfolio modeling: We assume a balanced portfolio where financial assets are invested 50% in Canadian equities and 50% in Canadian long-bonds, employing a buy-and-hold investment strategy where capital gains are realized only on withdrawal. We stochastically simulate future self-managed assets' annual net (after fees) real rates of return assuming that they are independently and identically normally distributed with a mean of 2% and standard deviation of 1%. These estimates were motivated by the 2017 projection assumption guidelines compiled by the Institut québécois de planification financière (IQPF) and the Financial Planning Standards Council (FPSC) (Bachand et al., 2017). See Appendix A for details.
- Inflation: We assume that inflation is independently and identically normally distributed from year to year with a mean of 2.0% (ibid) and a standard deviation of 0.7%, as calculated from historical inflation rates over the past twenty years (from 1996-2016). See Appendix A for details.

3.3 Tool of Analysis

This paper uses a longitudinal dynamic microsimulation model to project the financial consequences of alternative drawdown strategies within the Canadian retirement income system, while accounting for the uncertainty of future financial returns, inflation rates, and mortality. An important benefit of the microsimulation approach is that, while deterministic modeling examines a single potential future, individual microsimulation modeling simulates many (here, a million) independent futures following from a particular financial strategy. By tracking the income flows year-by-year as they interact with the financial market, inflation, and with the set of tax and benefits programs relevant for the individual, there is significant analytical flexibility to fully and clearly examine the financial consequences of alternative financial strategies.

Figure 1 illustrates the general simulation structure[14]. The top box of Figure 1 represents the input at the outset of each simulated lifetime. This information includes the subject individual's chosen drawdown

[12] Some examples include:
"Why women (especially) should delay taking CPP" https://www.theglobeandmail.com/globe-investor/retirement/retire-lifestyle/why-women-especially-should-delay-taking-cpp/article35945038/
"Asking Canadians to delay their CPP benefits? Good luck": https://www.theglobeandmail.com/globe-investor/retirement/retire-planning/asking-canadians-to-delay-cpp-retirement-benefits-good-luck/article33964757/
"Want your money to go further in retirement? Defer CPP until age 70": https://www.theglobeandmail.com/globe-investor/retirement/retire-planning/how-deferring-cpp-until-age-70-pays-off-for-retirees/article34209897/
"Why people hate the thought of deferring their CPP pension": https://www.theglobeandmail.com/globe-investor/retirement/retire-planning/cpp-pension-and-why-people-hate-to-wait-they-really-hate-it/article34244279/

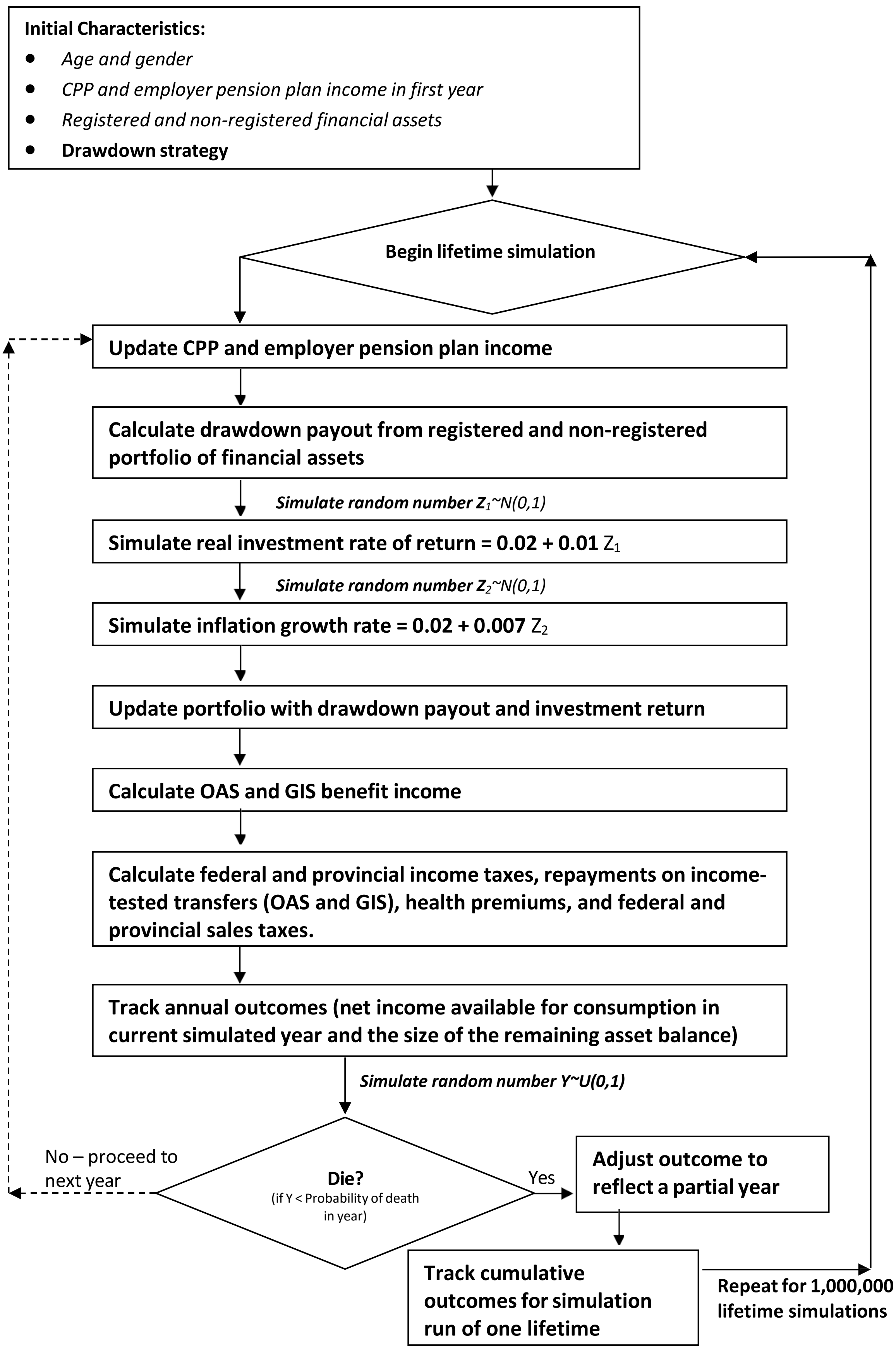
[13] We assume no individual-specific personal tax deductions – such as childcare or running a business, or credits for items such as charitable donations or medical expenses.

strategy, personal characteristics and financial resources in term of the tax assisted and non-tax assisted portfolio sizes at retirement, and the Canada Pension Plan and private (employer) pension benefit levels (defined in Section 3.1). The simulation first determines the various income sources that the person is eligible to receive during the coming year. Based on this, it next calculates the drawdown amount, which is then subtracted from the portfolio of financial assets, which accumulates according to the simulated asset returns, generating dividends, interest income, and a mix of realized and unrealized capital gains. The realization of capital gains, asset returns, and withdrawals from the portfolio all affect both the taxes payable and the composition of the portfolio heading into the next year. The individual pays all relevant income taxes at the end of the year, including repaying any government income-tested benefits that are “clawed back” as a result of the year’s income level, as well as taxes on capital gains.

Year by year, the model records the relevant annual information. This process continues until the individual has died within the year, which is determined by comparing a pseudo-random draw against standard mortality table rates. Once the individual dies, the model records the relevant information for the particular lifetime, and moves on to the next instantiation. Each run generates one million independent instantiations of the subject individual for the analysis that we report in this paper.

[14] This description summarizes that given in MacDonald et al. (2017).

Figure 1: General simulation structure of longitudinal dynamic microsimulation model (modified from Figure 1 in MacDonald et al. (2017))



The choice of a microsimulation approach reflects the challenges of using classical statistical and optimization techniques when evaluating alternative financial strategies. As discussed in MacDonald et al. (2013), more traditional optimization techniques encounter a variety of difficulties when modeling the realistic outcomes of alternative financial strategies, the most obvious challenge being its inability to incorporate the discontinuous functions and derivatives underlying the Canadian tax and transfer system. The second important advantage of the microsimulation approach is that it allows the analyst to track and examine the necessary simulated information in as much detail as is desired. So, while the traditional approach is often very limited in output, microsimulation opens the scope to evaluate the trade-offs between alternative financial strategies from a variety of perspectives – risks and returns such as annual or lifetime mean/medians for the various income flows, the quantiles associated with those income flows, or other distribution statistics such as "ruin", the probability that a particular government benefit is received, etc[15].

An important simplification in this analysis is that we do not impose the minimum RRSP withdrawal requirements. This requirement is irrelevant in the traditional academic literature on drawdown since, if the tax and transfer system is not incorporated, it can then be assumed (without mention) that non-desired minimum withdrawals are reinvested without any implications. In the real world, however, there are tax implications and future work will include enforcing the minimum withdrawals, reinvesting "overwithdrawals" and drawing them down later.

3.4 Outcome measures

We apply all of the relevant tax and benefit program rules, and strategy parameters, to derive the constant dollar net income available for consumption (referred to as "spendable income" to emphasize that it captures the comprehensive and integrated total net income).

A common statistic to measure dispersion is standard deviation, but there are challenges in its interpretation owing to the complexities of tax/transfer systems that lead to dispersions that are, primarily because of the system's intentional progressivity, not symmetric about the mean.

We calculate and report percentiles by age, therefore, focusing on the 10^{th}, median (50^{th}) and 90^{th}. For example, the 10^{th} "spendable income" percentile represents the value in a given year where 10% of the simulations produce spendable income below it, and 90% above it. The 10% and 90% percentiles represent the range within which 80% of the outcomes fall in that given year[16]. Section 4 predominantly employs graphics to demonstrate outcomes across the life-course, which includes the mean remaining portfolio balances by age.

We also calculate "probability of ruin" as a lifetime measure to summarize the risk exposure faced. The "probability of ruin" counts the number of simulated lifetimes in which the assets are depleted before the subject's death, and reports it as a proportion of total number of simulations.

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[15] Moreover, having access to the full slate of intervening variables considerably facilitates model validation - that is, ensuring that the model is doing what is supposed to be doing, which is particularly important in a tax/transfer system of this complexity.

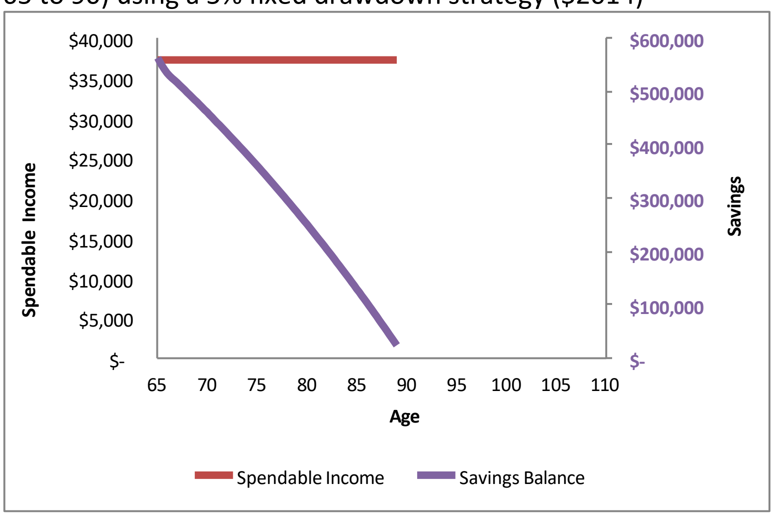
[16] Note that a simulated person who hits the 90^{th} percentile in a particular year does not signify, however, that that particular simulated future represents the 90^{th} percentile value in each and every year - some years that individual may lie above the 90^{th} percentile, and in others below it.

4 Results

We first investigate the implications of a conventional strategy that, according to a "deterministic" projection of the future, would exhaust assets by age 90. Figure 1 reports the deterministic net income available for consumption each year (spendable income on the left y-axis), and the financial savings (right y-axis). Each year, the spendable income is flat at $37,250 (inflation adjusted to year 2014), made-up of the following components:
Canada Pension Plan: $12,154
Old Age Security: $6.676
RRSP withdrawals: $25,100
Other savings: $502
TFSA withdrawals: $2,610
Taxes: $9,710

On the surface, this strategy is attractive to a person who, without a bequest motive, desires to use savings to fund retirement. The income stream is fixed, which facilitates financial planning and budgeting because the income does not fluctuate and is known in advance, and savings depletes nearly linearly until assets are exhausted at age 90. But there are three major problems with this conventional advice.

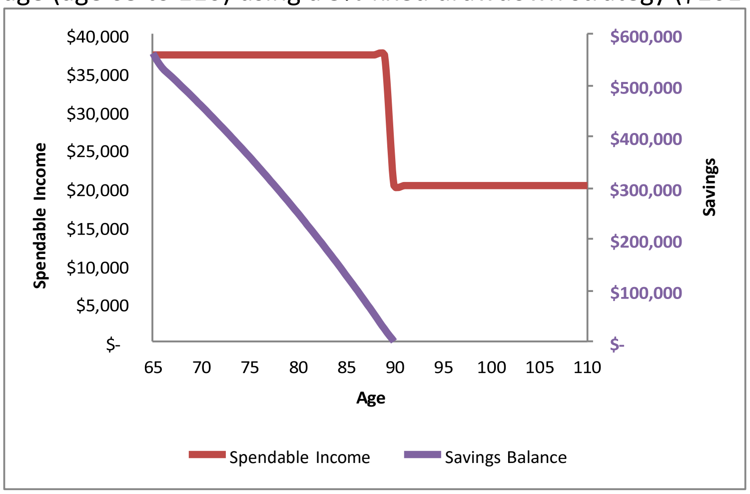
Figure 1: Deterministic projection of spendable income (left y-axis) and assets (right y-axis) by age (age 65 to 90) using a 5% fixed drawdown strategy ($2014)



First, what occurs if the subject lives past age 90? Figure 2 calculates the implications of living beyond age 90, thereby demonstrating the implications of longevity risk. If the subject lives beyond age 90, assets are depleted and spendable income drops by 46% to $20,250 (inflation adjusted to the year 2014), made-up of the following components:
Canada Pension Plan: $12,154
Old Age Security: $6.676
Guaranteed Income Supplement: $3,060
RRSP withdrawals: $0
Other savings: $0
TFSA withdrawals: $0
Taxes: $1,870

The income components that have changed since pre-age 90 are bolded. In addition to paying much lower levels of taxes, the subject begins receiving GIS benefits. Although the GIS program serves as excellent protection for lower-income Canadian seniors, the subject under consideration was initially categorized as an affluent Canadian according to wealth levels. By mismanaging the drawdown of savings, however, the net income is not only drastically decreased at a potentially vulnerable age, it creates a greater cost burden on working-Canadians. In this example, such a high net-worth Canadian male faces a 17% probability of running out of wealth, and a female of 31%. This becomes a societal concern if seniors who could otherwise fund their own retirement become dependent on federal and provincial income-tested programs that are paid for through tax revenue, particularly given the aging of the Canadian population.

Figure 2: Deterministic projection of spendable income (left y-axis) and financial assets (right y-axis) by age (age 65 to 110) using a 5% fixed drawdown strategy ($2014)

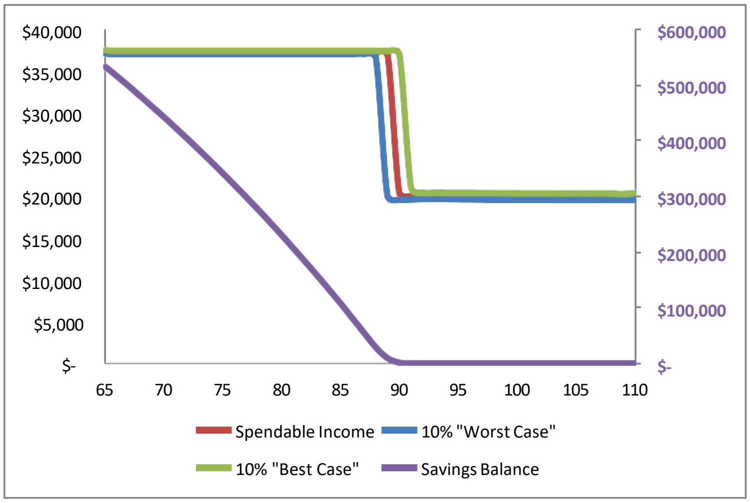


The third challenge with this conventional advice is that the probability of this future happening is zero. Although the assumption values are reasonable, the existence of risk by definition leads to any deterministic "one future" projection carrying a 0% probability. By simplifying the complexity of future financial outcomes into a single possible future, deterministic projections are valuable in that the underlying calculations are relatively simple and can often be done quickly, producing results that are easy to communicate. This simplicity can provide important insight into the basic workings of a complex system. But, by presenting only a single possible future that carries essentially a 0% probability, deterministic projections are very limited when understanding the implications of alternative financial strategies operating within a complex system in an unknown future.

Rather than examine one possible future, therefore, Figure 3 presents the ***distribution*** of spendable income across a ***million*** possible futures. By modeling the relationship between all of the components, our microsimulation approach captures the ongoing dynamics over time in alternative futures. Figure 3, and the figures thereafter, report on the range of future outcomes, which will help to unravel the implications of risk and understand the impact of risk mitigating financial strategies.

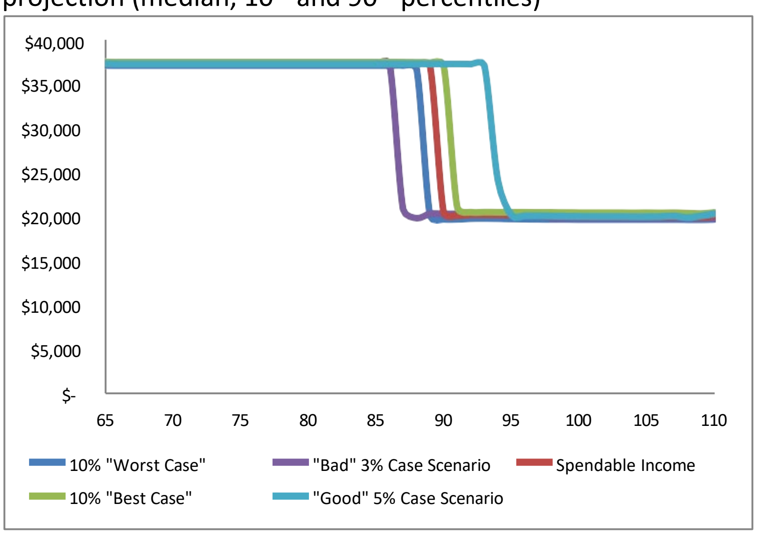
Figure 3 shows that there is a 10% probability of running out of savings before age 89, and a 10% probability after age 91. According to our assumptions, therefore, there is an 80% probability that assets will be depleted and spendable income reduced by 46% between ages 89 and 91.

Figure 3: Stochastic microsimulation projection of spendable income (median, 10th and 90th percentiles) and mean financial assets (right y-axis) by age (age 65 to 110) using a 5% fixed drawdown strategy ($2014)



A common deterministic technique to investigate the implications of financial market risks is to apply “stress testing”, where a handful of deterministic projections are performed at varying levels of financial market returns. Figure 4 illustrates a “good” and a “bad” case scenario where, rather than receive a fixed 4% after-fees nominal rate of return (2% real), the assets accumulate with a fixed 5% or 3% nominal return. In terms of our financial market model, this would be one standard deviation away from the mean. In the “bad” case scenario, the subject exhausts assets at age 87, and in the “good” case scenario at age 94. Although the “stress test” scenarios are not seemingly very extreme, these results fall far outside the 10% tails. The deterministic projection is clearly very sensitive to the assumed fixed rate of return, and seemingly reasonable “good” and “bad” case scenarios lead to extreme outcomes that are not probable.

Figure 4: Spendable income by age (age 65 to 110) using a 5% fixed drawdown strategy ($2014): deterministic projection with fixed 3% and 5% nominal rates of return, and stochastic microsimulation projection (median, 10th and 90th percentiles)



Treating the future deterministically, a strategy to manage the longevity risk of losing nearly half of spendable income beyond age 90 is to lower the withdrawals each year so that savings lasts to a very advanced age. According to the baseline deterministic assumptions, a fixed drawdown strategy of 3.3% would exactly exhaust savings by age 110 – as demonstrated in Figure 5.

Figure 5: Deterministic projection of spendable income (left y-axis) and financial assets (right y-axis) by age (age 65 to 110) using a 3.3% fixed drawdown strategy ($2014)

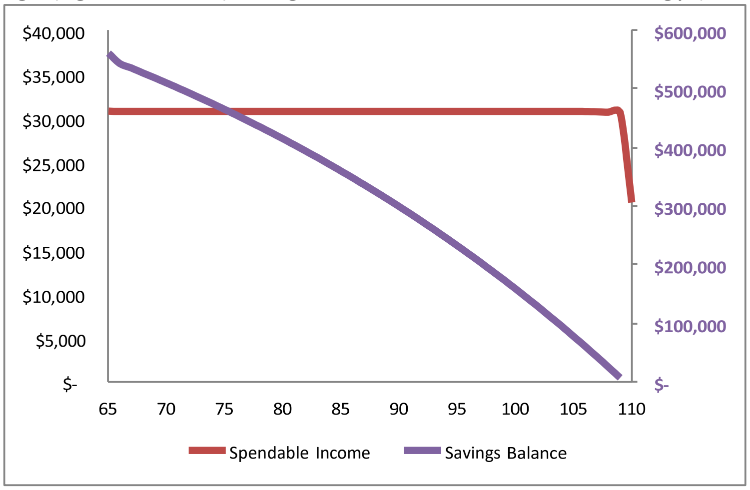


Figure 6 shows the implications of investment and inflation rate risk. Even in this very conservative strategy, there is a 10% risk of depleting assets and suffering a large decline in spendable income before age 107. The purpose of choosing a conservative drawdown strategy is to ensure adequate income later in life. Figure 6 demonstrates that even very low withdrawals do not protect against the risk of running out of savings at an advanced (and likely frail) age, which would likely result in unwelcomed changes when a steady income is likely critically important for survivors to cover ongoing and fixed health care costs.

Figure 6: Stochastic microsimulation projection of spendable income (median, 10^{th} and 90^{th} percentiles) (left y-axis) and mean financial assets (right y-axis) by age (age 65 to 110) using a 3.3% fixed drawdown strategy ($2014)

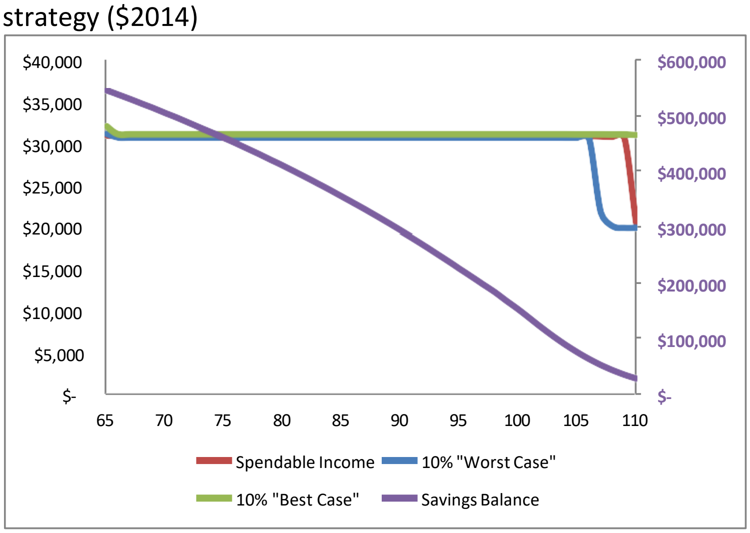
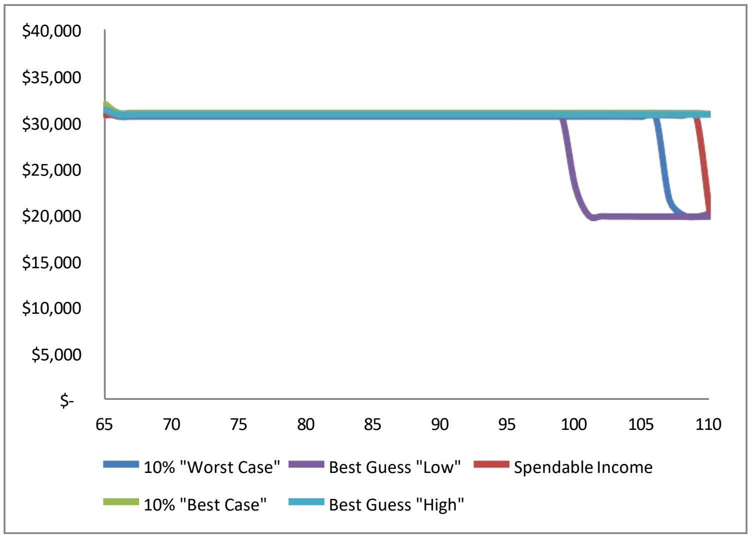


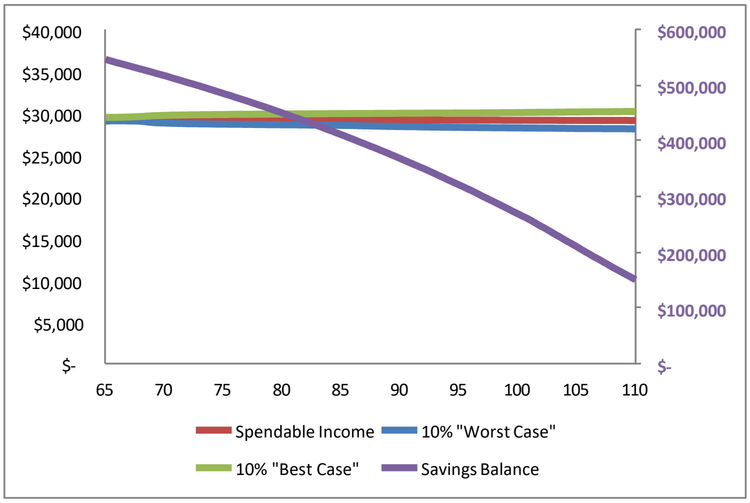
Figure 7 reports the deterministic “stress test” results, showing that assuming a fixed nominal 3% return leads to asset depletion by age 100 – a full seven years earlier than the 10% worst case. Again, these results support the challenge of depending on deterministic projections to understand the implications of risk.

Figure 7: Spendable income by age (age 65 to 110) using a 3.3% fixed drawdown strategy ($2014): stochastic microsimulation projection (median, 10th and 90th percentiles), and deterministic projection with fixed 3% and 5% nominal rates of return



Ensuring that savings are not exhausted can be achieved through a risk-mitigating drawdown strategy – a variable dynamic drawdown strategy, described in Section 3.2. Figure 8 presents the results of the variable dynamic drawdown strategy, where payments are adjusted each year to reflect financial market performance (see Section 3.2 for precise formulas). The spendable income by age on the left y-axis of Figure 8 shows that, even in the 10% worst and best simulations, the payments continue to be reasonably steady within a narrow corridor. This financial strategy is designed so that assets (graphed on the right y-axis) are never exhausted prior to the intended age (110), which ensures that net income does not suddenly drop. In addition, Section 2 discusses the empirical evidence showing that seniors prefer not to spend savings out of concern for later-life financial shocks – this strategy fits this preference by ensuring an ongoing contingency reserve of available funds for unanticipated financial shocks.

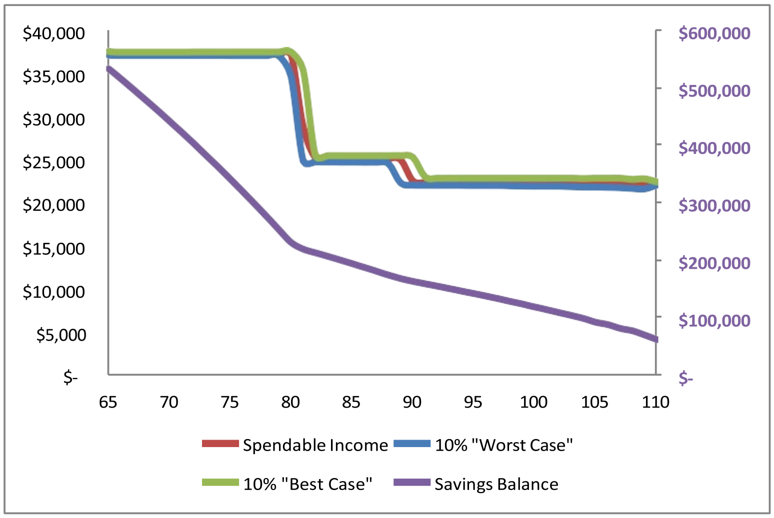
Figure 8: Stochastic microsimulation projection of spendable income (median, 10th and 90th percentiles) (left y-axis) and mean financial assets (right y-axis) by age (age 65 to 110) using a variable drawdown strategy ($2014)



The drawdown strategy in Figure 8 is extremely conservative, and may not fit the preferences of retiring Canadians who desire higher levels of flexible income in the earlier part of retirement, and a lower level of secure income during the later portion. Figure 9 shows the results of an alternative, hybrid, strategy designed to have higher spendable income during the younger (and generally disability-free) retirement years, and a stable income thereafter. This is achieved by allocating 50% of the RRSP savings to the variable drawdown strategy, and employing a fixed drawdown strategy that targets the initial income of the original Figure 2 for the remaining assets (which is calculated to be 5% fixed withdrawals for the non-registered wealth and TFSAs, and 7.1% fixed withdrawals of the other 50% of RRSP savings).

In Figure 9, the income begins at the original spendable income of $37,250 (inflation adjusted to year 2014) from Figure 2. Unlike in the original strategy, however, the first drop of 32% ($37,250 to $25,200) occurs at approximately age 82 when the RRSP savings allocated to the fixed drawdown strategy is exhausted. A further drop of 6% to $22,500 occurs when the TFSA and non-registered wealth is exhausted at age 90 (with a 10% probability of before age 89 and a 10% probability of happening after age 91). Thereafter, this strategy provides a relatively secure level of retirement income of $22,500 during later years and a continuing pool of accessible funds.

Figure 9: Stochastic microsimulation projection of spendable income (median, 10th and 90th percentiles) and mean financial assets (right y-axis) by age (age 65 to 110) using a hybrid of variable and fixed drawdown strategies ($2014)



But, there is a financial strategy that generates better financial outcomes. By delaying CPP take-up to age 70 and using RRSP wealth to bridge the gap, Figure 10 shows the direct improvement to both the retirement income expectation and risk exposure. Targeting the same initial income, there is an initial drop of 23% to $28,800 at a slightly earlier age of 80, and the second drop at age 90 of an additional 7% to $26,000.

Figure 10: Stochastic microsimulation projection of spendable income (median, 10th and 90th percentiles) and mean financial assets (right y-axis) by age (age 65 to 110) using a hybrid of variable and fixed drawdown strategies, plus delay CPP take-up to age 70 (and using savings to bridge the gap) ($2014)

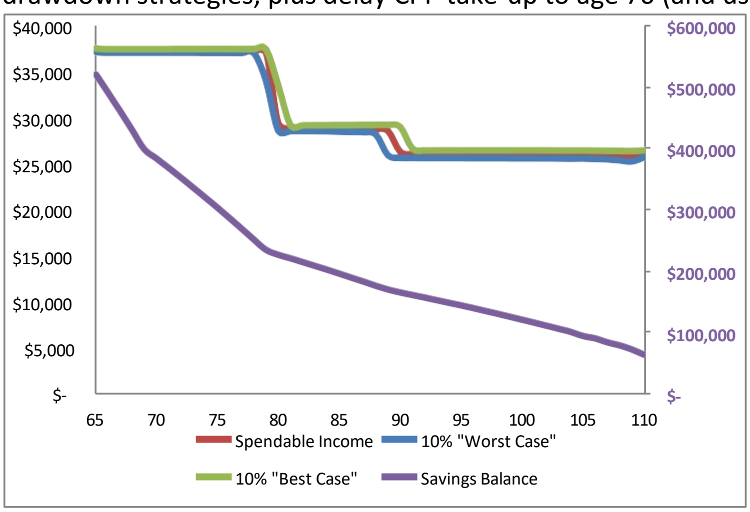


Figure 11 superimposes these two strategies. The delay CPP strategy achieves equal or higher median income levels over the entire retirement, except for a small differential between ages 80 and 82. The same applies for the risk measures: the income at the 10th and 90th percentiles achieve higher income levels in the "delay CPP" strategy, other than between ages 80 and 82. Overall, delaying CPP take-up to age 70 achieves significantly higher income levels and reduced risk. The primary trade-off in choosing to delay CPP is an anticipated reduced RRSP balance, as additional funds are directed to bridge the CPP delay.

Figure 11: Figure 9 graphed against Figure 10

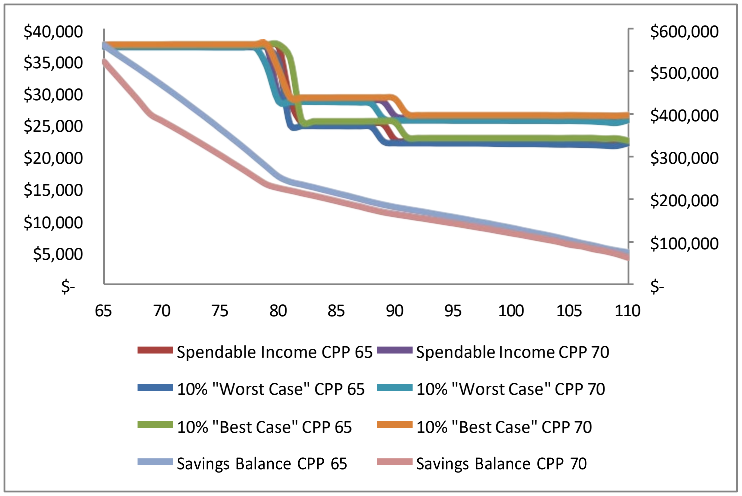


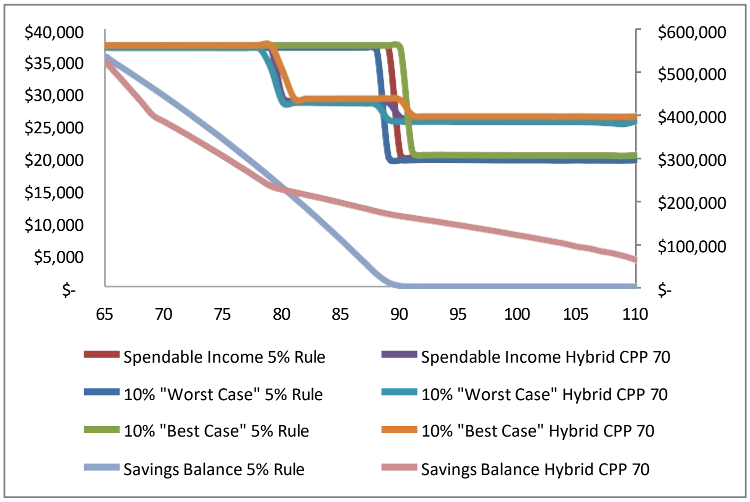
Figure 12 compares the risk-reducing strategy in Figure 10 to the original strategy in Figure 3. In exchange for an anticipated 25% drop in net income between ages 80 and 90, the risk-reducing strategy:

- Safeguards that net spendable income is unlikely to drop by no more than 30% over retirement (compared to the 46% drop, which would likely occur between ages 89 and 91 in the original strategy).
- Guarantees an additional $5,100 in CPP income until death, with no financial market risk, inflation risk, or longevity risk[17].
- Ensures a continuing pool of accessible funds to protect against unanticipated financial shocks and other needs.
- Produces no dependence on the GIS program.

Considering the strong empirical evidence that seniors desire greater later-life financial security, but may not want to lose control over accessible funds , the advantages of the risk-reducing strategy fits the needs and preferences of seniors, while also protecting Canadian tax-payers from the financial risk and burdens of affluent Canadian seniors mismanaging their financial wealth and becoming eligible for additional senior social transfers.

[17] The CPP inflation-indexation is guaranteed provided the CPP continues to be affordable. The Chief Actuary's report projects that the CPP program is sustainable for at least the next 75 years (Canada, 2015)

Figure 12: Figure 3 graphed against Figure 10



5. Discussion – Bridging Theory and Practice

The choice of the variable drawdown strategy in this analysis was motivated by the prior literature, that found that this strategy significantly reduces the risk of exhausting savings. However, the variable drawdown is simply the automatized annual recalculation of withdrawals based on market performance. In the "real" world, this type of risk management already occurs when financial strategies are regularly revisited and revised according to market performance. Often done by a financial planner, such reviews will typically account for other critical considerations as well, such as inflation, life expectancy and any other material changes in a person's circumstances and preferences. Therefore, this research strongly supports the practice of proactively monitoring and reviewing financial strategies[18].

The challenge, however, is that people who depend on financial planners may not return year after year to revisit their strategies. In this case, the ongoing risks cannot be managed, and the dangers of deterministic projections continue to apply. Given this obstacle, it remains important that advisors who rely on deterministic projections qualify their limitations and proactively inform clients of alternatives to a "set and forget" strategy.

This research also highlights the value of point-in-time decisions that can protect people from risk – for example, at what age to start collecting C/QPP and OAS, when to tap into home equity (and by how much), whether to buy medical insurance or self-manage health expenses and others (see Section 2). Given that deterministic projections cannot appropriately evaluate the value of such risk-reducing

[18] Another advantage of periodic reviews and recalculations is that, although not explored in this paper, they help to mitigate the consequences of incorrect assumptions (e.g., model risk). For example, as the anticipated long-term financial returns are adjusted based on emerging experience, the recalculation will gradually reduce the levels of withdrawals if the original assumption was too high, or increase them if the original assumption was too low. Acknowledging that long-term future mean rates of returns are difficult to predict, and there is a wide range of opinion on the matter, periodic reviews help protect against this risk.

financial choices, the individual or advisor has greater responsibilities to use alternative (softer) approaches to explore risks in alternative futures. Factors to consider include the anticipated level and sources of secure income, tax implications, health and longevity expectations, and anticipated informal support from family. Future research by the authors aims to produce an understandable, evidence-based framework to help Canadians with a range of financial and personal circumstances to determine the best ages to start collecting CPP benefits. This work is intended to be made available to the public.

6. Conclusion

This paper investigates the implications of adopting deterministic projections that ignore the risks associated with retirement financial planning. Using an affluent Canadian case example, we first present a conventional approach to retirement financial planning. We then introduce mortality, financial market and inflation rate risk, and explore the implications. Finally, we investigate alternative drawdown strategies where the individual targets the same financial income flows but reduce overall risk by adjusting, throughout their lifetimes, those factors that they can control.

Employing the 2017 Projection Assumption Guidelines compiled by the IQPF and the FPSC (Bachand et al., 2017), this analysis is carried out using a longitudinal dynamic micro-simulation model that explicitly models the Canadian retirement income system as it projects the financial consequences of alternative drawdown strategies, while accounting for the uncertainty of future financial returns, inflation rates, and mortality. This model simulates the consequences of alternative financial strategies by generating distributions for rich realistic sets of outcomes that would flow from using those strategies. The primary metrics for this paper include the median real consumption ("spendable income") by age, and the dispersion as measured by the 90% and 10% percentiles. We also calculate the mean savings levels and the probability of exhausting savings ("ruin").

Financial planning involves significant exogenous risks that cannot be controlled. Individuals can, however, improve their financial outcomes and reduce overall risk by adjusting the factors they can control – such as adjusting drawdown rates according to asset performance and delaying CPP uptake. The financial outcomes of the risk-mitigating strategies offers improved protection from risk for our case example, while also better fitting the preferences of seniors in general who, as suggested by empirical evidence, prefer to maintain control over accessible funds while having secure later-life income.

Retirement financial security has become critically important with an aging population. As the shift away from defined benefit pension plans and traditional informal services provided within the family continues, it is becoming increasingly important (and necessary) for workers to be engaged in their own retirement financial planning. This study finds that not only do deterministic projections ignore critical financial risks that can derail retirement financial security, they also are unable to model or assess the value of risk-reducing financial strategies. This means they are also unable to identify valuable financial planning strategies that address a retiring Canadian's unique needs while managing risk and maximizing income.

The research also found that conventional "stress testing" – a common industry technique intended to help inform risk implications – leads to extreme and improbable outcomes. The validity of stress testing is an area for future research, although these preliminary results are not promising. A more sophisticated strategy – likely involving a Monte Carlo approach that models many possible future scenarios – seems to be required.

The belief among some in the financial planning world that deterministic analyses are "good enough" is problematic from both the individual and societal perspectives. Risk implications may lead to hardships for seniors at advanced and vulnerable ages, which is also a societal concern. If the otherwise affluent Canadian retirees mismanage their assets by treating the future deterministically, this will put greater stress on families and communities – in addition to an already-burdened healthcare system – and will create an even greater dependency on federal and provincial income-tested programs.

One practical way to overcome some of these limitations is to regularly monitor and review the financial strategies – either by the person themselves, or by their advisor. Reflecting market performance, inflation, life expectancy and any other material changes in a person's circumstances in the strategy will help better manage risk and increase the likelihood of the individual achieving his/her goals. However, since these strategies often aren't revisited, the limitations of deterministic projections should be clearly explained. This study also points to the importance of approaching key decision opportunities – such as the age to begin collecting CPP benefit payments – appreciating that the implications of risk that cannot be appropriately captured in a deterministically projected cashflow analysis.

Although this analysis demonstrates the positive impact of risk-reducing strategies, making financial choices to improve risk exposure and better match personal preferences is an individual exercise. This research supports the critical importance of using sophisticated models that bring together a person's full financial circumstances, within the environment of taxes, transfers and risks in which we operate.

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Appendix A: Calculating Portfolio Assets Rates of Return and Inflation Model Parameters

The average long-term rate estimates underlying the financial market and inflation models in this study are motivated by the 2017 projection assumption guidelines compiled by the Institut québécois de planification financière (IQPF) and the Financial Planning Standards Council (FPSC) (Bachand et al., 2017) (referred to as "guidelines" in this section).

We stochastically simulate future annual inflation rates of return assuming that they are independently and identically normally distributed with mean (μ_k) and standard deviation (σ_k). We assume μ_k to be 2% according to the guidelines (ibid). The Bank of Canada has targeted a stable 2% inflation rate since 1991, and we estimate the standard deviation (σ_k) using historical inflation data since 1996[19]:

$$\bar{\sigma}_k = \sqrt{\frac{1}{2016-1996+1-1}\sum_{t=1996}^{2016}(k_t - \bar{\mu}_k)^2} = 0.7\%$$

We stochastically simulate future self-managed assets' annual real rates of return assuming that they are independently and identically normally distributed with mean (μ_p) and standard deviation (σ_p). To estimate μ_p and σ_p , we follow the 2017 projection assumption guidelines (Bachand et al., 2017), where the recommended net nominal return after fees for a balanced portfolio is approximately 4% (page 11), leading to a 2% net real return (μ_p = 2%). We set the standard deviation σ_p = 1%, which was loosely motivated by the guidelines recommendation that projections can deviate by 0.5% and still be within the guidelines. By setting σ_p = 1%, there is some alignment between the recommendation of being within half of a standard deviation (0.5%) to be within the guidelines. (For the standard normal distribution, 38% of the observations lie within one standard deviation of the mean, and 68% of the observations lie within two standard deviations of the mean.)

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[19] All-items Consumer Price Index Source: CANSIM V41690973

We set σ_p = 1% owing to the alignment, but more importantly owing to the empirical support. Assuming a balance portfolio of Canadian equities and long-bonds, we can estimate our financial market model using historical data compiled by the Canadian Institute of Actuaries 2017 report on Canadian financial statistics (CIA, 2017). The underlying data sources are:

- Bond returns:
    - Yield-to-maturity on Government of Canada marketable bonds (10+ years) from 1936-2016
    - CANSIM I: B14013; CANSIM II: V122487
- Stock returns[20]:
    - Total return on Canadian Common Stock from 1936 - 2016
    - Prices:
        - Urquhart & Buckley H641 December 1936–December 1946 (Corporate Composite)
        - CANSIM B4202 (TSE Corporates) December 1946–December 1956
        - TSX Total Return Index December 1956–December 2016
    - Dividend Yield, Annual Averages:
        - Urquhart & Buckley H617 January 1951–December 1955
        - CANSIM V122628 January 1956–December 2016

To estimate μ_p and σ_p using this historical data, we first obtain historical real rates of return for our assumed portfolio assets (50% equities and 50% risk-free assets, assuming a buy-and-hold strategy for long-term bonds) from the above listed historical data sources. Letting:

- b_t^n represent the bond total nominal rate of return,
- b_t^r represent the bond total real rate of return,
- s_t^n represent the stock total nominal rate of return,
- s_t^r represent the stock total real rate of return, and
- k_t represent the rate of consumer price inflation,

between times *t* and *t+1*. Hence:

$$b_t^r = \frac{1+\ {}_t^n}{1+k_t} - 1,$$

[20] Data source summary are taken from CIA (2017).

and

$$s_t^r = \frac{1+s_t^n}{1+k_t} - 1.$$

We estimate mean (μ_p):

$$\mu_p = 0.5\frac{1}{2016-1936+1}\sum_{t=1936}^{2016} s^r + 0.5\frac{1}{2016-1936+1}\sum_{t=1936}^{2016} b^r = 4.84\%$$

We assume a buy-and-hold strategy for the bond investment, thereby making it a risk-free asset. We estimate the annual fixed total return ($\overline{b^r}$) using the average real yield-to-maturity on long-term bonds. Hence, we estimate the standard deviation of the portfolio (σ_p):

$$\overline{\sigma}_p = \sqrt{\frac{1}{(2016-1936+1)-1}\sum_{t=1936}^{2016}(s_t^r + \overline{b^r} - \overline{\mu}_p)^2} = 8.22\%$$

The challenge of employing historical data is that the anticipated future returns are much lower than historical experience in Canada as well as other industrialized countries (Reinhart and Rogoff, 2009; Guay and Jean, 2013; King and Low, 2014), and therefore fixing a future long-term net rate at the lower 2% according to the guidelines is justifiable. But it is unclear how to model the future dispersion of financial market returns (σ_p), particularly when the projected long-term rate is not set to its historical average (as is the case in this study). Employing more recent data (2000-2016), we find that it produces a lower μ_p of 3.85% (2.6% after fees) and an even higher σ_p of 8.66%. These results lend support to setting μ_p to 2% as specified in the guidelines, but it also suggests that dropping the projected long-term real net return to 2% should not necessarily impact the standard deviation assumption. But a 8.66% standard deviation is quite large and, if normally distributed, would generate negative real rate of return over 40% of the simulations. On the other hand, between 1936 and 2016, real rates of return on the assumed portfolio were negative in 25% of the years, and 22% of the years between 2000 and 2016.

Rather than match the standard deviation, therefore, we set the standard deviation so that there is a 25% probability of generating a negative real rate of return. If *X* is independently and identically normally distributed with mean (μ_p) and standard deviation (σ_p) then, according to the standard normal probability table:

$$P\left(\frac{X-\mu_p}{\sigma_p} \le -1.96\right) = 25\%$$

$$P\left(X \le -1.96\sigma_p + \mu_p\right) = 25\%$$

If the goal is to set σ_p so that negative real returns occur in 25% of simulations then

$$P\left(X \le 0\right) = 25\%$$

and

$$-1.96\sigma_p + \mu \ = 0$$

$$\sigma_p = \mu_p / 1.96$$

For $\mu_p = 2\%$, $\sigma_p = 1.02\%$, matching the original 1% assumption.